$$x_{\min} = m\, c_1 / (m - Y).$$

Parallel results for auctions in which bidders are competing to purchase are available by replacing costs by values and the parameter m by $-m$. For details on all of these results, see Rothkopf [1969].

3. Subsidizing the Less Efficient of Two Potential Suppliers

We will refer throughout the paper to the more efficient bidder as the "first-line" bidder, and normalize cost by setting a first-line bidder's project cost equal to 1. It bears mention that the normalization factor is thus implicitly known to the analyst, but unknown to any bidder.

Consider the situation that would result if the bidtaker offers publicly to subsidize the designated (less efficient) bidder, d, whose cost is c_d (> 1). Let the subsidy take the following form: a subsidy set at s implies that, should a designated bidder make the low bid, he will win the contract but be paid 100 s% more than his bid. In the bidding model, this implies that d's cost will be only $c = c_d/(1 + s)$. Then both bidders will bid as if bidder d's cost is in fact c. The impact of this subsidy policy is summarized in Tables 1 and 2; readers not interested in the formulas behind the tables can skip the next paragraph.

Using the results developed in Rothkopf [1969] and described above, in equilibrium the expected payment to the winning bidder, not counting any subsidy, will be $mc/(m\text{-}Y)$, and $Y = \{m(1\text{-}c) + [m^2(1\text{-}c)^2 + 4c]^{1/2}\}/2$. Since the probability that bidder d wins is given by $Y/(1+Y)$, the expected amount of the subsidy is $(c_d\text{-}c)Y/(1+Y)$, and the bidtaker's overall expected cost is given by $Z = mc/(y\text{-}m) + (c_d - c)Y/(1+Y)$.

Consider first Table 1. Throughout this table, $u = 0.12$, that is, the standard deviation of the cost-estimating error of each bidder is approximately 12% of the unknown true mean [u is shown in column (2)]. The table presents banks of rows, with the rows in the first bank identified in column (1) by 1.1–1.9, the next by 2.1–2.8, etc. A given bank fixes the relative cost of the designated bidder (c_d), at 3 times that of the first-line bidder in bank 1, and successively closer to no cost disadvantage [c_d is shown in column (3)].

The first row in each bank provides equilibrium outcomes when no subsidy is granted, and each successive row increases the subsidy, tabulated in column (5). For example, row 1.2 presents the case in which a 50% subsidy allows a designated bidder whose cost is 3 times that of a first-line bidder to bid as if his cost were only 2 times as high (this is the entry in column (4), c). The final row in each bank of Table 1 fully subsidizes the designated bidder, allowing him to compete on an even basis with the first-line bidder.

A private-sector bidtaker interested in minimizing expected project cost will choose a subsidy based on the entries in column (6), labeled *Pvt $*, which is our shorthand for the equilibrium level of expected project cost (Z above). We will refer to the expected-project-cost-minimizing subsidy as the *optimal subsidy*; the row corresponding to the optimal subsidy in each bank has been emphasized via boldface.

Column (7) shows a public-sector bidtaker's objective, *Pub $*, which is discussed in section 3.2. We refer to the subsidy level minimizing that objective as the *efficient subsidy*, and emphasize its row via italicized boldface.[9] Column (8), E:*Pay*, is the expected payment to the winning bidder, excluding any subsidy. The probability of a designated bidder winning, Pr:*Ineff*, is in column (9); in each case where the designated bidder is fully subsidized, of course, this probability reaches 0.5. This probability is multiplied by the average subsidy payment to obtain the unconditional expectation of the subsidy, E:*Subs*, in column (10). The final two columns report derivatives of the objectives in columns (6) and (7) with respect to the subsidy *s*, shown in column (5).

The selected entries in Table 1 should make clear the pattern of the underlying continuous process. Without a subsidy, the designated bidder presents little competition for the first-line bidder. Accordingly, the first-line bidder rationally bids a significant mark-up on his cost estimate. For example, on average, the bidtaker pays [column (8)] 3.05 times this bidder's cost, to this bidder 87% of the time [= 1 - column (9)], when his cost advantage is 1-to-3 (row 1.1), and 2.04 times his cost 84% of the time when his cost advantage is 1-to-2 (row 2.1).

If the choice were simply between no subsidy and a full subsidy (the last row in each bank), in each case shown here the full subsidy that puts the competitors on an even footing leads to a lower expected project cost, including the subsidy. For example, fully subsidizing a designated bidder with a 20% cost disadvantage (bank 4) reduces the bidtaker's expected cost from 1.26 to 1.22.

In general, though, fully subsidizing the designated bidder is not optimal. Further reductions in project cost can be obtained by switching to an intermediate subsidy, which only partially levels the playing field. For example, it is optimal to compensate for a 40% cost disadvantage with a subsidy of nearly 30% (row 3.5). As the cost disadvantage is reduced in successive banks, the

[9] While we refer (conventionally) to a private-sector bidtaker as seeking to minimize expected cost, this is based upon analysis of this auction in isolation. Circumstances will compel some private-sector bidtakers to conduct auctions less myopically (cf. Harstad [1993]), if, say, the number of bidders responds to expected profitability.

optimal subsidy as a percentage of full subsidy gradually increases from 68% in bank 1 to 71% in bank 2 to between 75% and 80% in the lowest four banks.

The basic intuition is this: it does not pay the first-line bidder to bid aggressively enough to shut out the designated bidder; driving his own winning probability to 1 reduces too substantially his profitability in the events he would already win. Without any subsidy, the expected cost to the bidtaker is slightly above the designated bidder's cost. When the bidtaker offers a subsidy to the designated bidder, he is forcing the first-line bidder to bid more aggressively; the savings this produces more than offsets the expected cost of the subsidy.

3.1. Varying Bidders' Cost Uncertainties

Table 2 keeps the cost disadvantage of the designated bidder constant at 20%, but it varies u, the standard-deviation-to-mean ratio. When u is small compared with the cost disadvantage (top three banks), the best course for the bidtaker is to offer the designated bidder a subsidy that brings his effective cost from 120% of the first-line cost to just over 104% of it. However, as u increases (larger bidder uncertainties) the optimal subsidy increases until in the two lowest banks it is in his interest to give the designated bidder an advantage--lowering his effective cost to 96% (row 6.5) and 89% (row 7.5) of the first-line bidder's cost. Note, however, that while the subsidy is dramatic, the estimating uncertainty is so large that even with the large subsidy the designated bidder's chance of winning the auction [column (9)] has gone only from 47% to 51% (in bank 6) or 52% (in bank 7). Also, the objective function improvement beyond the effect of fully subsidizing is small. Situations requiring a subsidy exceeding the disadvantage do not seem to be a robust phenomenon; for example, we find in section 4.2 that, for these parameters, the presence of another first-line bidder makes partial subsidizing optimal again.

3.2. Allocative Efficiency Considerations

A public-sector bidtaker may well be concerned with allocative efficiency.[10] If so, the appropriate objective considers the allocative efficiency gain associated with lower project costs implying lower tax rates, and hence less distortion. However, this is offset by another consideration: whenever an inefficient bidder wins, a socially wasteful level of the underlying cost of contract

[10] We assume that the public-sector concern with revenue stems from an underlying efficiency concern. That is, it is not a desire to raise more revenue regardless of the consequences. There may be benefits of subsidies beyond the purely economic considerations presented here; such benefits would presumably argue for larger subsidies. Societal costs of subsidies beyond the purely economic costs we incorporate, if significant, would imply recommendations of lower levels of subsidies, but presumably would not undo the qualitative argument for subsidy at some positive level.

fulfillment results. The relative weight that would be placed on these two considerations depends upon the marginal excess burden of the tax system.[11]

The expected amount of the inefficiency associated with the choice of an inefficient supplier is given by $P^*(c_d\text{-}1)Y/(1+Y)$. Thus, the overall objective of such an agency should be to minimize $Z_e(K) = KZ + P^*(c_dY+1)/(1+Y)$, where K is a factor reflecting the marginal efficiency impact of raising money for the government and the second term in the expression for Z_e is the expected amount of allocative inefficiency generated by the possibility that the designated bidder will win the contract. If K equals one, raising a dollar in taxes would cost a dollar in economic activity. Throughout the paper, we will use the quite conservative estimate of 4/3 for this excess burden: reducing project cost by \$3 million and reducing taxes by the same amount will increase national output by \$1 million as lower taxes distort taxpayers' behavior less.[12] $Z_e(4/3)$ [column (7)] and its derivative with respect to s [column (12)] are also displayed in Tables 1 and 2.

The *efficient* subsidy (the level that minimizes this measure of economic inefficiency) cannot be greater than the optimal subsidy (that minimizes the bidtaker's total payments), but it is generally a substantial fraction of it. Table 1 begins with an efficient subsidy which is about 82% of the optimal subsidy, and it remains above 75%, increasing slightly in the lowest two banks. Like the best subsidy for the bidtaker, it can (in seemingly unusual situations, cf. Bank 7 of Table 2) call for a subsidy beyond equality. In Table 2, where the efficiency difference remains moderate at 20% and the estimating accuracy is varied, the

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[11] The treatment of subsidizing domestic bidders competing against foreign firms in McAfee and McMillan [1989] treats government taxation as having no excess burden. This is an all-too-common neglect in microeconomic policy analyses.

[12] This is one of the more conservative estimates that Ballard, Shoven and Whalley [1985] offer for the aggregate US federal tax system, for conservative assumptions about key supply elasticities. Ballard [1991] (private communication) indicated that calculations following the 1986 tax law changes were insignificantly altered. Judd [19@@] argues that the static model used substantially underestimates tax distortions in a dynamic economy. Feldstein [1995] estimates the marginal excess burden of the 1993 tax increase to be at least 4. Moreover, the estimates are for the marginal excess burden of an incremental change in all tax rates; if reduced project cost could be directed to cutting the most distortive tax, a higher efficiency gain would result. One factor could work in the other direction: if only a fraction of project cost reduction went into lower tax rates, with the remainder going into additional government spending that had no similar marginal efficiency gain, then the efficiency gain resulting from lower project cost would be reduced. It is possible to conceive a scenario in which all of the project cost reduction went into added government spending of no social value whatsoever, in which case subsidies would have no efficiency advantages (note that this is presumably a scenario in which economists' advice has absolutely no influence on policy). The point is that treating the payment to the bidtaker as a pure transfer can only be correct for a knife-edge case in which an exactly balancing proportion of project cost reduction goes into added wasteful spending. For an extensive discussion of the magnitude of K and the appropriateness of its use in efficiency calculations such as this one, see Rothkopf and Harstad [1990].

efficient subsidy ranges from about 84% down to 72% of the optimal subsidy, again increasing slightly as the lowest banks of Table 2 are reached.

4. Subsidizing with More than Two Suppliers

This section uses the n-bidder version of the multiplicative-strategy model to explore similar issues to those just covered in the context of two bidders. The n-bidder model is discussed in section 5 of Rothkopf [1969]. It does not have a known analytic equilibrium solution; we rely on numerical methods. Since in the model bidders' strategies are scalars, this is not as daunting as it would be if they were general functions. The 1969 paper has some numerical results that were excerpted from a more thorough set of tables contained in an internal Shell Development Company report, Rothkopf et.al. [1966], that recently Shell has graciously agreed to release, and we have further extended.

This approach, while adequate, does not provide the precision that the use of the analytic formulas in the two-bidder analysis did. Instead of adjusting the subsidy arbitrarily finely to find precise minima, only subsidies that change the situation from one tabulated case to another are considered. We then approximate minima by inspection and interpolation.

4.1. A Single Designated Bidder

Tables 3-6 illustrate subsidy impacts with three or more bidders. A new column (1), labeled *Com*, reports the composition of the set of bidders as a ratio $f:d$ of f first-line bidders to d designated bidders. Thus, throughout Table 3, two first-line bidders compete with a single designated bidder. Fixing the same moderate estimating uncertainty [u = 0.12, column (2)] as in Table 1, successive banks of Table 3 consider reduced efficiency differentials [c_d, column (3)].

There is no need to present situations with as large an efficiency disadvantage as the first two banks of Table 1; even with as little as a 12% cost differential, competition between the two first-line bidders keeps bids low enough to leave an unsubsidized designated bidder completely out of the running. A subsidy attempting to overcome a cost differential as large as 40% (bank 1 of Table 3) will never reduce project costs enough to overcome the subsidy payments: a 25% subsidy is not big enough to bring the designated bidder into play, and a subsidy big enough to have even a slight impact is already too expensive. No subsidy, or any ineffective subsidy (up to 25%) is optimal and efficient.

However, a private-sector bidtaker would choose to subsidize a designated bidder with only a 30% cost disadvantage (bank 2) in order to add some competition for his two first-line bidders. But a subsidy has to exceed 16%

before it has any effect, and it quickly becomes costly, so that it is not optimal to subsidize enough to let the designated bidder win much more than 3% of the time. Indeed, the optimal subsidy only serves to increase the bidding aggressiveness of the two first-line bidders by less than 0.05%. Even this generates so much inefficiency that a public-sector bidtaker prefers not to subsidize.

Subsidizing to get a third bidder involved is always called for in the more moderate cost disadvantages of 20% or less. These last four banks of Table 3 correspond to the same parameters as the last four banks of Table 1. The presence of the second first-line bidder reduces the optimal subsidy from 15% to about 12% (c_d = 1.2), 7.7% to 6.8% (1.1) and 3.9% to closer to 2.9% (1.05), and reduces the efficient subsidy from 11% to about 8%, 5.8% to near 3.8%, and 2.9% to more like 1.9%. These reductions seem surprisingly slight, since the Table 1 subsidies strengthened the only competitor to the single first-line bidder. When c_d = 1.2, the single first-line bidder was bidding 13% less aggressively than each of the two first-line bidders now bid when they face no effective competition. Yet some effective competition is worth subsidizing to a large fraction of the extent called for when it was the only effective competition.

4.2. Varying Bidders' Cost Uncertainties

The two first-line bidders compete more aggressively in equilibrium when their estimating accuracy is higher: they need to correct less for the winner's curse effect, and face a rival with a cost estimate on average closer to their own. This lessens the bidtaker's incentive to subsidize the one designated bidder, as illustrated in Table 4. This table returns to the benchmark of a 20% cost advantage, and repeats as its second bank the third bank of the last table: u = 0.12. The first bank represents only about half as much uncertainty, and now it does not pay either a private- or a public-sector bidtaker to use subsidies to bring the third bidder into effective competition: both objectives are minimized in row 1.1, where he has no chance of winning. Thus, the cost and efficiency numbers for that row also apply to a 0 subsidy. Notice the importance of the second first-line bidder to this result: the same disadvantage and uncertainty parameters for the two-bidder case are in bank 2 of Table 2, where the efficient subsidy is almost 15% and the optimal subsidy 11.7%.

The greater uncertainty displayed in the lowest three banks of Table 4 does call for partial subsidization to bring in the third bidder. Again, the presence of the second first-line bidder reduces its extent: for u = 0.12, the optimal subsidy is reduced from 15.5% to about 12% and the efficient subsidy from 11.4% to near 8%. More striking is that the extent of subsidization increases only slightly as the estimating uncertainty increases. The last bank of Table 4, with the standard deviation of the estimating error over half of the mean cost, goes beyond the *u* values at the bottom of Table 2, which were enough to create oversubsidization.

4.3. Additional First-line Bidders

Having more than two first-line bidders competing with a single designated bidder magnifies the effects of adding a second first-line bidder that were just discussed, further reducing the extent of subsidization. Bank 1 in Table 5 repeats the numbers from the third bank of Table 3 (which were also Table 4's second bank). Now Table 5's bank 2 adds a 3rd first-line bidder [*Com* in column (1) becomes 3:1]. It no longer pays a private- or a public-sector bidtaker to subsidize the designated bidder in order to add a fourth effective competitor. (Any row with a 0 probability of an inefficient outcome [column (9)] also indicates the project costs of a 0 subsidy; the positive subsidy is shown to give a rough idea of how large a subsidy remains ineffective.)

The lowest three banks of Table 5 consider a more nearly competitive designated bidder, with only a 10% cost disadvantage. Bank 3 repeats the fourth bank of Table 3. Going from three bidders to four (i.e., to bank 4 of Table 5) reduces the optimal subsidy, and removes the government's incentive to subsidize. Finally (bank 5), a fifth bidder makes subsidizing privately unprofitable.

4.4. One First-line Bidder

Next, we turn to the situation in which there are more than two bidders all but one of whom is disadvantaged. This models the situation in which one bidder has a natural advantage—perhaps incumbency, natural monopoly or critical mass—that all others lack. The model grants the same subsidy to any subsidized bidder; the one first-line bidder is facing stiffer competition in that he must outbid the most competitive of these bidders to win.

The top bank in Table 6 considers three bidders, two of whom have a 40% cost disadvantage. The optimal subsidy is about 27%, slightly smaller than the 29.5% subsidy that was optimal for a single competitor with the same 40% differential (third bank of Table 1).[13] The efficient subsidy is also lessened only slightly by the additional designated bidder, to near 20% from 22.7%. This 40% cost differential was so large that neither type of bidtaker chose to subsidize a

[13] The precision shown in columns (4) and (5) of Table 6 is misleading: we only have data for values of c such that $10/c$ is an integer.

single designated bidder when two first-line bidders competed (top bank of Table 3).

The more moderate cost disadvantage of 20% (with $u = 0.12$) is treated in Table 6's middle bank, for contrast with Table 1, bank 4, and Table 3, bank 3. Partial subsidization is chosen in all cases. With one bidder of each type, Table 1 shows optimal and efficient subsidies of 15.1% and 11.4%. When the third competitor is a first-line bidder, the selected subsidies drop to near 12% and 8%. Not surprisingly, here, with the added competitor also disadvantaged, the results are intermediate: an optimal subsidy of about 14%, and efficient subsidy just above 9%. The lowest bank of Table 6 introduces further competition (maintaining our benchmarks: $c_d = 1.2$, $u = 0.12$): preferred subsidies with 5 competitors, a single first-line and four designated bidders, are about 12.8% and 8%. Thus, starting from a base of one bidder of each type, subsidies are reduced by approximately the same extent by adding either one more first-line bidder, or three more designated bidders.

Finally, Table 7 completes the circuit by varying bidders' uncertainty, u, returning to one first-line bidder and two bidders with a 20% cost disadvantage. The top bank repeats Table 6's second bank, and lower banks increase u. The additional uncertainty does not appear to create significant variation in the efficient subsidy, even though large enough uncertainty eventually shows up (lowest bank) in an optimal subsidy (pure expected-cost-minimization) which closely approximates putting the two designated bidders on an even footing with the one first-line bidder. Note though, that this 20% optimal subsidy is still small compared with the over-subsidization that was both optimal and efficient with slightly less uncertainty and only one designated bidder (Table 2's lowest bank). As expected, the preferred subsidies here are lower than the two-bidder case, but higher than those where the added competitor was a second first-line bidder (Table 4). Interestingly, a government buying from three bidders who face dramatic cost uncertainties ($u = 0.52$, meaning the standard deviation of cost estimates is over half the mean) selects about the same subsidy level for either composition (the pair of bidders being either type; compare the last banks of Tables 4 and 7).

Subsidizing two of three bidders incurs the most serious inefficiencies, as even small subsidies are claimed by winners quite frequently, especially with greater cost uncertainties. So it is not surprising that the ratio of the optimal subsidy to the efficient subsidy is generally as low in this scenario as in any of the foregoing; the lowest bank of Table 7 is the only place we have seen a government bidtaker prefer to subsidize, but to less than half the extent a cost-minimizing bidtaker would.

5. Long-Run Costs and Benefits of Subsidies

The previous sections have shown that the increased competitiveness of first-line bidder behavior in response to subsidies for designated bidders can more than make up for the cost of the subsidy, including the allocative cost of occasionally awarding a project to an inefficient supplier. However, straightforward microeconomic analysis implies that subsidies reduce the profitability of first-line bidders and raise designated bidders' expected profits.

This raises the question of whether, in the long run, subsidies might be poor economic policy, because they might make entry into the supplier industry disproportionately likely to be entry of designated firms, rather than efficient bidders. (The recent FCC radio spectrum auctions offer a striking example: new entrants have almost all been designated bidders, and a suspicion of at least some technological disadvantage seems apt.) Even if the form of entry into the industry is an endogenous choice, the decreased profitability of first-line bidders, partly to the benefit of designated bidders, could reduce the incentive to incur capital or other incremental costs associated with upgrading a bidder's capabilities to the first-line level of efficiency.

We illustrate the tradeoff between long-run and short-run effects with a simple but not pathological example in which the relationship between the level of subsidies and the probability of entry of each type of bidder is ad hoc. The underlying calculations come from the benchmark case, in which $u = 0.12$ (standard deviation of estimating errors is 12% of cost) and $c_d = 1.2$ (designated bidders at a 20% cost disadvantage).

Suppose that a government will undertake a procurement project every year, and evaluates results in terms of economic efficiency (with a tax distortion factor of 4/3, as above), converting to present values via a 4% discount rate. In year 1, a single first-line firm and a single designated firm bid. Throughout this section, we maintain the assumption that bidders follow the equilibrium strategies discussed above for a single auction, rather than attempting the sort of "Folk theorem" tacit collusion which might yield other equilibria of a repeated game. The government has a choice between a credible commitment for all time to a "no subsidy" policy N, and a credible commitment for all time to a policy S which invokes the efficient subsidy for designated bidders.

So long as there were only two bidders in the previous year, we assume that each period has a chance of an entry of another bidder. To keep things simple, we assume that entry of a third bidder, of whatever type, forestalls further entry. There is no exit.[14] If the government adopts policy N, a second first-line bidder enters with probability p_{NF}, or (exclusively) a second designated bidder enters with probability p_{ND}, in any period with only two incumbent bidders. Should the government instead adopt policy S, the entry probability of a first-line bidder decreases to p_{SF}, while the entry probability of a designated bidder increases to p_{SD}, reflecting the altered profitability of bidding.

Policy S begins with a subsidy of 11.4% (Table 1, row 4.3). After entry of a first-line bidder, policy S shifts to a subsidy of 8.1% (Table 3, row 2.1), and policy N continues no subsidy (so the presence of the designated bidder becomes irrelevant). After entry of a designated bidder, S shifts to a 9.2% subsidy, and we treat N as shifting to a 0.8% subsidy, which is the smallest subsidy for which we have a figure for the expected social project costs (Table 6, bank 2). Consequently, the expected (undiscounted) efficiency costs in a given year are:

	Policy N (no subsidies):	Policy S (subsidies):
Before Entry:	1.7464	1. 7114
After Entry by		
First-Line Bidder:	1.4815	1.4814
Designated Bidder:	1.7002	1.6727

By construction, for any given collection of bidders, policy S is superior. Thus, whenever entry is not sharply and differentially affected by the government's policy, S will remain superior: efficiency will call for subsidies.

One scenario is that there is simply a single established firm, which will not get any competitors on an even footing; so $p_{NF} = p_{SF} = 0$. However, it is currently challenged by a single designated bidder, and only a subsidies policy will create a chance of another designated bidder entering: $p_{SD} > p_{ND} = 0$. This possibility enhances the benefits of the subsidies policy S offers. While the subsidy reduces yearly project cost before entry by 2.005%, the discounted infinite stream of project costs is reduced from 45.407 for N to 43.937 for S if $p_{SD} = 0.05$, a 3.235% reduction due to subsidies.

This is not the only long-run possibility, however. Initially suppose that $p_{NF} = p_{ND} = 0.1$, while $p_{SF} = 0.075$ and $p_{SD} = 0.125$. These numbers have the

[14] Also for simplicity, there is no type switching. In some situations, this neglects an advantage of subsidies: the designated bidder who wins more frequently today due to subsidies may acquire the experience necessary to become a first-line competitor tomorrow. This is an argument advanced by affirmative action proponents.

governmental policy shift leave the overall entry probability unaffected, which may be unlikely. However, it allows a focus on simply shifting the type of entry that does occur: the subsidy makes an entrant far more likely to be a designated bidder. For these probabilities, policy N has a lower expected cost in year 8 (1.6235, undiscounted) and beyond, than does policy S (1.6241). The present discounted social cost of the infinite stream of projects is 42.036 for policy N, 42.104 for S. If instead of a shift of 2.5 percentage points, entry was shifted only 2 percentage points (p_{SF} = 0.08 and p_{SD} = 0.12), S would retain its initial advantage, with a discounted sum of 42.0003.[15]

Suppose that an infinite horizon falls beyond a government's purview, so that all years beyond some political horizon are discounted completely. Then, in a more extreme example, a six-year horizon is long enough to prefer policy N. An example sets $p_{NF} = p_{ND} = 0.1$, $p_{SF} = 0.01$, and $p_{SD} = 0.19$. Then the undiscounted six-year sums of social cost are 10.119 for N and 10.157 for S. Again, when the effects on entry are less volatile, subsidies can gain. Even over a horizon where entry has made subsidies less attractive on narrow economic grounds, proponents of subsidies will be able to argue that their economic costs are negligible.

6. Imperfectly Informed Bidtakers

The model presented assumes that both first-line and designated bidders can look at announced subsidy policies and know c, the designated bidder's as-if cost disadvantage incorporating the subsidy. This implies that both types of bidders know the underlying extent of disadvantage, c_d. However, the analytics require no presumption that the bidtaker knows c_d. In some instances, as when a private-sector developer repeatedly auctions contracts to install sewer pipes in new subdivisions, experience may lead the bidtaker to know this variable nearly as well as the bidders. However, the case in which the bidtaker is notably less well informed is surely at least as common. In particular, a public-sector agency which procures a variety of goods and services from a variety of industries may have some information about designated bidders' restricted access to capital markets, but likely cannot be nearly as well informed about cost differentials in

[15] These calculations assume that the need to procure the project recurs each year with certainty. If procurement continuing is less than certain, then it takes a more dramatic impact of subsidies on entry to reverse the short-run impact. For example, suppose the project is procured each year with probability 0.9455 if it was procured the previous year, and 0 otherwise. This has the same impact as raising the discount rate to 10%. Then the same (0.1, 0.1) probabilities for policy N yield a discounted sum of 18.070. Now a larger shift than before, reducing the probability of policy S yielding another first-line bidder by 3 percentage points to (0.07, 0.13) remains superior, at 18.051, while a shift of 3.5 points to (0.065,0.135) becomes inferior, at a discounted sum of 18.086.

each of the industries it deals with as are bidders whose livelihood depends upon knowing industry conditions.

So it is sensible to ask whether a subsidy chosen by a bidtaker with somewhat incorrect beliefs about c_d might do more harm than good. Our results show sufficient robustness that cautious choice of a subsidy is quite likely to be better than no subsidy at all.

As an illustration, consider the situations presented in Table 1. Throughout, if either a private-sector or a public-sector bidtaker chose the optimal subsidy for a believed level of c_d which was *less* than the true c_d, the result was still preferable to the no-subsidy outcome. That is, the subsidy level shown in bold in one bank of numbers in Table 1 still reduces private cost [column (6)] in any bank of rows higher up the table, relative to the no-subsidy private cost shown in the first row of the bank. Similarly, the subsidy level shown in italics in one bank of numbers in Table 1 still reduces public cost [column (7)] in any bank of rows higher up the table, relative to the no-subsidy public cost shown in the first row of the bank.

A small overestimate of the native disadvantage c_d also leads to subsidy levels that are better than not subsidizing, though the chosen subsidies are too large, and in cases where the actual c_d was less than 3/4 of the believed c_d, can lead to giving the designated bidder so great an advantage that he wins over half the time. For example, suppose a bidtaker believed designated bidders to be at a 20% disadvantage, and accordingly chose a 15% (private sector) or 11% (public sector) subsidy. Table 8 illustrates (with the benchmark of Table 1, $u = 0.12$). A 15% (private sector, top half of table) or 11% (public sector, bottom half of table) is too large if the underlying cost disadvantage is only 15% ($c_d = 1.15$, middle column of figures). But it achieves an expected cost of 1.1946 [1.6592, public sector] rather than the 1.2180 [1.6778] that would result from no subsidy. As the last two rows in each half of the table show, selecting the subsidy level that would have been optimal had the cost differential been 20% still captures about 90% of the gains available to a perfectly informed subsidy setter.

That represents considerable robustness, though the fraction of benefits captured starts to fall sharply as the beliefs about cost differentials become quite inaccurate. When an actual cost differential of 10% is perceived as 20%, 80% or more of the benefits of subsidizing are foregone by setting the subsidy in accordance with the overly stark perception. Somewhat sharper misperceptions finally reach the point where the liberal subsidy no longer improves the bidtaker's objective: at about $c_d = 1.094$, the 15% [11%, public] subsidy achieves no savings relative to no subsidy.

It is at least as plausible, moreover, that the bidtaker may have incorrect beliefs about the amount of estimating uncertainty bidders face. Glancing back at Table 2, a bidtaker who chose the optimal or efficient subsidy associated with a given value of u would still prefer that subsidy to no subsidy for any larger value of u (further down the table). A bidtaker who believed a very high value of u could be worse off with the preferred subsidy if in fact u were dramatically less than believed. But the bidtaker cannot go very far wrong by assuming a moderate value like 0.28, as Table 9 shows. The format of Table 8 is replicated (the 17.2% and 12.8% subsidies are the choices from Table 2's fourth bank), and almost all of the possible gains to full information setting of subsidies are maintained when the bidtaker in fact overestimated considerably the extent of bidders' uncertainty. The general conclusion, then, is that a bidtaker who is aware that his knowledge of relative cost differentials and of the extent of bidders' uncertainty is imperfect ought not be dissuaded from all subsidies. A conservative choice of subsidy is better than no subsidy.

7. Conclusions

In the FCC's "Regional Narrowband" auction of radio spectrum rights,

> The extra revenue the government earned from unsubsidized winning bidders, such as PageMart, more than offset the subsidy to the designated bidders. Far from being a giveaway, affirmative action bidding preferences induced competition that prevented established firms from buying the airwaves at substantial discounts to their own valuations. (Ayres and Cramton [1996], p. 403.)

Our results provide an analytical foundation for the finding in those auctions that a bidtaker can benefit from a policy of subsidizing disadvantaged competitors. The theoretical model explicates how general the phenomenon is, and why it works. (While our model dealt with procurement auctions, parallel results can be obtained for disbursement auctions.) It also provides useful comparative static information which is unlikely to be gleaned from empirical databases.

Briefly summarizing, public-sector bidtakers, because of concern for the misallocation of resources that results when a less efficient provider wins, always prefer smaller subsidies than are optimal for private-sector bidtakers, but generally achieve an efficient allocation of resources with a nonnegligible reduction in the subsidized competitors' cost disadvantage. Often this efficient subsidy is two-thirds to four-fifths of the optimal, i.e., cost-minimizing subsidy, which is still only a partial subsidy, save for extreme circumstances. Preferred subsidies are found to become larger when the bidtaker needs more competition from the disadvantaged bidders, which includes cases where the bidders face greater uncertainty, and when the cost disparity is large. Additional designated bidders reduce subsidies or remove the case for them, and additional first-line bidders make this effect even more pronounced. Bidtakers who are imperfectly informed about relative cost disadvantages and estimating accuracies in supplier industries nonetheless ought to adopt a conservative subsidy policy, as subsidies that are better suited to different conditions nonetheless generally outperform zero subsidies.

Over time, a policy of subsidizing designated bidders may increase their number, which enhances its benefits. However, if it prevents entry of first-line bidders that might otherwise have occurred, this reduces but probably does not overturn the benefits of subsidizing.

We close by pointing to distinctly different economic consequences of two affirmative action policies that often go hand-in-hand: subsidies and set-asides for designated bidders. In the FCC's Regional Narrowband auction, minority-owned firms were favored by [a] setting aside two of the six licenses in each geographic region for bidding only by them, [b] subsidizing their bids on all licenses, and [c] subsidizing their bids on the set-aside licenses to a greater degree. Ayres and Cramton [1996] treat this as a single policy and analyze its effects. However, the revenue gains from the policy that they find when they trace through the bidding are the result of [b] above, when it is viewed as a separable policy. The evidence from that auction is entirely consistent with the hypothesis that revenue would have been even higher had the FCC set up subsidies for minority bidders at the two levels used, but then allowed all bidder to bid on any of the licenses. The major impact of [a] above was to prevent unsubsidized bidders from competing for some of the licenses. Indeed, it was because minority bidders were not satisfied with competing for the set-aside licenses that they generated more aggressive bidding by unsubsidized firms.

A subsidy policy can only probabilistically affect the proportion of contracts awarded to designated bidders, and detailed knowledge of industry conditions is needed to be able to calculate in advance the probabilities that will result from any particular level of subsidy. One interpretation of the FCC's policies is that they interpreted a Congressional mandate as meaning that they had to guarantee that minority bidders would win at least one-third of the licenses. Set-asides make such guarantees straightforward, while they would add considerable complications to an auction where all bidders can in principle compete for all assets.[16]

[16] It is not impossible to accomplish a guarantee without prespecified set-asides. For example, the FCC could conduct a multi-stage progressive auction with the rule that the tentatively winning bids at the end of any round are that combination of bids which yield the greatest revenue among all combinations which award at least the requisite number of licenses to designated bidders. If the high bids award too few licenses to them, then this rule implies that the bids by designated bidders which come closest to the high bids on some licenses are declared tentative winners. It is also true that in a sequence of auctions, the FCC could adjust subsidy levels to achieve approximately some target penetration level of preferred bidders.

While a complete model of entry into multiple-unit auctions, with subsidies and set-asides, by potential first-line and designated bidders, is beyond the scope of this paper, it is clear that set-asides remove or at least reduce the incentive for designated bidders to compete with first-line bidders, while subsidies enhance this incentive. It is the natural best response of first-line bidders to this competition, more aggressive bidding, that often serves to more than cover the costs of subsidies. This reaction by first-line bidders is completely ignored in the usual accounts of the cost of subsidy policies; in fact, their cost may often be negative, for reasons we have explained.

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Table 1

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
ID	*u*	*Cd*	*c*	*s*	*Pvt $*	*Pub $*	E:*Pay*	Pr:*Ineff*	E:*Subs*	d:*Pvt $*	:*Pub $*
1.1	0.12	3	3.000	0%	3.045	4.320	3.05	0.13	0.00	-1.84	-2.34
1.2	0.12	3	2.000	50%	2.375	3.494	2.04	0.16	0.33	-0.93	-1.07
1.3	0.12	3	1.500	100%	2.056	3.183	1.54	0.22	0.51	-0.37	-0.19
1.4	0.12	3	***1.421***	***111%***	2.021	***3.172***	***1.47***	***0.24***	***0.55***	*-0.26*	***0.00***
1.5	0.12	3	1.400	114%	2.013	3.173	1.45	0.24	0.57	-0.23	0.06
1.6	0.12	3	1.300	131%	1.990	3.208	1.35	0.28	0.64	-0.05	0.37
1.7	0.12	3	**1.273**	**136%**	**1.988**	3.228	**1.33**	**0.29**	**0.66**	**0.00**	0.47
1.8	0.12	3	1.200	150%	2.000	3.319	1.26	0.33	0.74	0.17	0.79
1.9	0.12	3	1.000	200%	2.222	3.963	1.11	0.50	1.11	0.65	1.61
2.1	0.12	2	2.000	0%	2.040	2.884	2.04	0.16	0.00	-1.33	-1.65
2.2	0.12	2	1.500	33%	1.715	2.507	1.54	0.22	0.17	-0.66	-0.66
2.3	0.12	2	1.300	54%	1.615	2.430	1.35	0.28	0.26	-0.31	-0.08
2.4	0.12	2	***1.278***	***57%***	1.607	***2.429***	***1.33***	***0.29***	***0.28***	*-0.26*	***0.00***
2.5	0.12	2	1.200	67%	1.590	2.445	1.26	0.33	0.33	-0.08	0.32
2.6	0.12	2	**1.172**	**71%**	**1.588**	2.461	**1.24**	**0.34**	**0.35**	**0.00**	0.45
2.7	0.12	2	1.100	82%	1.600	2.532	1.18	0.40	0.42	0.22	0.82
2.8	0.12	2	1.000	100%	1.667	2.722	1.11	0.50	0.56	0.49	1.21
3.1	0.12	1.4	1.400	0%	1.447	2.027	1.45	0.24	0.00	-0.851	-0.98
3.2	0.12	1.4	1.200	16.7%	1.343	1.921	1.26	0.33	0.08	-0.389	-0.28
3.3	0.12	1.4	***1.141***	***22.7%***	1.325	***1.913***	***1.21***	***0.36***	***0.11***	*-0.209*	***0.00***
3.4	0.12	1.4	1.100	27.3%	1.319	1.918	1.18	0.40	0.14	-0.068	0.21
3.5	0.12	1.4	**1.081**	**29.5%**	**1.318**	1.924	**1.16**	**0.42**	**0.15**	**0.000**	0.31
3.6	0.12	1.4	1.050	33.3%	1.320	1.939	1.14	0.45	0.18	0.111	0.47
3.7	0.12	1.4	1.000	40%	1.333	1.978	1.11	0.50	0.22	0.278	0.69
4.1	0.12	1.2	1.200	0%	1.2609	1.746	1.261	0.326	0.000	-0.56	-0.61
4.2	0.12	1.2	1.100	9.1%	1.2249	1.713	1.178	0.398	0.047	-0.23	-0.12
4.3	0.12	1.2	***1.077***	***11.4%***	1.2207	***1.711***	***1.161***	***0.419***	***0.060***	*-0.14*	***0.00***
4.4	0.12	1.2	1.050	14.3%	1.2183	1.714	1.142	0.446	0.076	-0.03	0.15
4.5	0.12	1.2	**1.042**	**15.1%**	**1.2181**	1.715	**1.137**	**0.454**	**0.081**	**0.00**	0.19
4.6	0.12	1.2	1.000	20%	1.2222	1.730	1.111	0.500	0.111	0.16	0.40
5.1	0.12	1.1	1.100	0%	1.1780	1.6105	1.178	0.40	0.000	-0.33	-0.34
5.2	0.12	1.1	1.060	3.8%	1.1687	1.6019	1.149	0.44	0.020	-0.16	-0.11
5.3	0.12	1.1	***1.040***	***5.8%***	1.1664	***1.6008***	***1.135***	***0.46***	***0.031***	*-0.08*	***0.00***
5.4	0.12	1.1	1.030	6.8%	1.1658	1.6011	1.129	0.47	0.037	-0.04	0.06
5.5	0.12	1.1	**1.022**	**7.7%**	**1.1656**	1.6018	**1.124**	**0.48**	**0.042**	**0.00**	0.10
5.6	0.12	1.1	1.000	10%	1.1667	1.6056	1.111	0.50	0.056	0.09	0.22
6.1	0.12	1.05	1.0500	0%	1.1419	1.5448	1.142	0.446	0.000	-0.17	-0.17
6.2	0.12	1.05	1.0300	1.94%	1.1394	1.5426	1.129	0.467	0.011	-0.08	-0.06
6.3	0.12	1.05	***1.0203***	***2.91%***	1.1388	***1.5423***	***1.123***	***0.477***	***0.016***	*-0.04*	***0.00***
6.4	0.12	1.05	**1.0109**	**3.87%**	**1.1386**	1.5426	**1.117**	**0.488**	**0.021**	**0.00**	0.05
6.5	0.12	1.05	1.0050	4.48%	1.1387	1.5430	1.114	0.494	0.025	0.03	0.09
6.6	0.12	1.05	1.0000	5%	1.1389	1.5435	1.111	0.500	0.028	0.05	0.12
7.1	0.12	1.02	1.0200	0%	1.12272	1.50651	1.123	0.478	0.000	-0.070	-0.070
7.2	0.12	1.02	1.0100	0.99%	1.12225	1.50611	1.117	0.489	0.005	-0.025	-0.011
7.3	0.12	1.02	***1.0082***	***1.17%***	1.1222	***1.50610***	***1.116***	***0.491***	***0.006***	*-0.017*	***0.000***
7.4	0.12	1.02	1.0050	1.49%	1.12218	1.50613	1.114	0.494	0.008	-0.003	0.018
7.5	0.12	1.02	**1.0044**	**1.55%**	**1.12218**	1.50614	**1.114**	**0.495**	**0.009**	**0.000**	0.022
7.6	0.12	1.02	1.0000	2%	1.12222	1.50630	1.111	0.500	0.011	0.019	0.047

Table 2

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
ID	*u*	c_d	*c*	*s*	*Pvt $*	*Pub $*	E:*Pay*	Pr:*Ineff*	E:*Subs*	d:*Pvt $*	:*Pub $*
1.1	0.025	1.2	1.2000	0%	1.203	1.625	1.203	0.106	0.000	-1.04	-1.29
1.2	0.025	1.2	1.1000	9.1%	1.124	1.533	1.105	0.174	0.019	-0.66	-0.64
1.3	0.025	1.2	1.0800	11.1%	1.112	1.523	1.086	0.202	0.026	-0.52	-0.37
1.4	0.025	1.2	***1.0609***	***13.1%***	1.1033	***1.519***	***1.068***	***0.240***	***0.036***	***-0.33***	***0.00***
1.5	0.025	1.2	1.0500	14.3%	1.100	1.521	1.058	0.268	0.043	-0.17	0.29
1.6	0.025	1.2	**1.0402**	**15.4%**	**1.099**	1.526	**1.049**	**0.299**	**0.050**	**0.00**	0.63
1.7	0.025	1.2	1.0400	15.4%	1.099	1.526	1.049	0.300	0.050	0.00	0.64
1.8	0.025	1.2	1.0200	17.6%	1.105	1.550	1.033	0.385	0.072	0.49	1.53
1.9	0.025	1.2	1.0000	20%	1.122	1.597	1.020	0.500	0.102	1.00	2.35
2.1	0.062	1.2	1.200	0%	1.2171	1.6665	1.217	0.219	0.000	-0.82	-0.95
2.2	0.062	1.2	1.100	9.1%	1.1601	1.6088	1.125	0.310	0.035	-0.40	-0.27
2.3	0.062	1.2	1.080	11.1%	1.1534	1.6055	1.108	0.338	0.045	-0.27	-0.06
2.4	0.062	1.2	***1.074***	***11.7%***	1.1519	***1.6053***	***1.104***	***0.347***	***0.048***	*-0.23*	***0.00***
2.5	0.062	1.2	1.050	14.3%	1.1482	1.6090	1.085	0.390	0.064	-0.05	0.29
2.6	0.062	1.2	**1.045**	**14.9%**	**1.1481**	1.6110	**1.081**	**0.401**	**0.067**	**0.00**	0.36
2.7	0.062	1.2	1.020	17.6%	1.1510	1.6253	1.064	0.453	0.087	0.21	0.67
2.8	0.062	1.2	1.000	20%	1.1579	1.6439	1.053	0.500	0.105	0.37	0.89
3.1	0.12	1.2	1.200	0%	1.2609	1.7464	1.261	0.326	0.000	-0.56	-0.61
3.2	0.12	1.2	1.080	11.1%	1.2211	1.7114	1.163	0.416	0.058	*-0.15*	-0.02
3.3	0.12	1.2	***1.077***	***11.4%***	1.2207	***1.7114***	***1.161***	***0.419***	***0.060***	-0.14	***0.00***
3.4	0.12	1.2	1.050	14.3%	1.2183	1.7135	1.142	0.446	0.076	-0.03	0.15
3.5	0.12	1.2	**1.042**	**15.1%**	**1.2181**	1.7149	**1.137**	**0.454**	**0.081**	**0.00**	0.19
3.6	0.12	1.2	1.040	15.4%	1.2182	1.7155	1.135	0.456	0.083	0.01	0.20
3.7	0.12	1.2	1.000	20%	1.2222	1.7296	1.111	0.500	0.111	0.16	0.40
4.1	0.28	1.2	1.200	0%	1.4843	2.0659	1.484	0.434	0.000	-0.22	-0.23
4.2	0.28	1.2	1.100	9.1%	1.4700	2.0529	1.405	0.465	0.065	-0.09	-0.06
4.3	0.28	1.2	***1.064***	***12.8%***	1.4673	***2.0518***	***1.378***	***0.477***	***0.089***	*-0.05*	***0.00***
4.4	0.28	1.2	1.050	14.3%	1.4667	2.0520	1.368	0.482	0.099	-0.03	0.02
4.5	0.28	1.2	**1.024**	**17.2%**	**1.4663**	2.0533	**1.349**	**0.491**	**0.117**	**0.00**	0.06
4.6	0.28	1.2	1.000	20%	1.4667	2.0556	1.333	0.500	0.133	0.03	0.10
5.1	0.36	1.2	1.2000	0%	1.6634	2.3089	1.663	0.455	0.000	-0.14	-0.14
5.2	0.36	1.2	1.1000	9.1%	1.6537	2.3002	1.579	0.476	0.075	-0.07	-0.05
5.3	0.36	1.2	***1.0465***	***14.7%***	1.6509	***2.2989***	***1.536***	***0.489***	***0.115***	*-0.03*	***0.00***
5.4	0.36	1.2	1.0200	17.6%	1.6502	2.2992	1.515	0.495	0.135	-0.01	0.02
5.5	0.36	1.2	**1.0008**	**19.9%**	**1.6500**	2.3000	**1.501**	**0.500**	**0.149**	**0.00**	0.04
5.6	0.36	1.2	1.0000	20%	1.6500	2.3000	1.500	0.500	0.150	0.00	0.04
6.1	0.43	1.2	1.2000	0%	1.84458	2.55269	1.845	0.4663	0.000	-0.10	-0.09
6.2	0.43	1.2	1.0250	17.1%	1.83400	2.54441	1.688	0.4954	0.146	-0.03	-0.01
6.3	0.43	1.2	***1.0146***	***18.3%***	1.83369	***2.54437***	***1.679***	***0.4973***	***0.155***	*-0.02*	***0.00***
6.4	0.43	1.2	1.0000	20.0%	1.83333	2.54444	1.667	0.5000	0.167	-0.02	0.01
6.5	0.43	1.2	**0.9585**	**25.2%**	**1.83290**	2.54546	**1.633**	**0.5079**	**0.200**	**0.00**	0.03
6.6	0.43	1.2	0.9000	33.3%	1.83396	2.54922	1.587	0.5197	0.247	0.02	0.06
7.1	0.48	1.2	1.2000	0.0%	2.02659	2.79671	2.027	0.473	0.000	-0.063	-0.054
7.2	0.48	1.2	1.0000	20.0%	2.01667	2.78889	1.833	0.500	0.183	-0.031	-0.016
7.3	0.48	1.2	***0.9531***	***25.9%***	2.01524	***2.78842***	***1.791***	***0.507***	***0.224***	*-0.018*	***0.000***
7.4	0.48	1.2	0.8000	50.0%	2.01640	2.79512	1.662	0.533	0.354	0.024	0.051
7.5	0.48	1.2	**0.8917**	**34.6%**	**2.01446**	2.78937	**1.737**	**0.517**	**0.277**	**0.000**	0.021
7.6	0.48	1.2	0.7000	71.4%	2.02410	2.80909	1.587	0.551	0.437	0.045	0.076

Table 3

ID	(1) Com	(2) u	(3) c_d	(4) c	(5) s	(6) *Pvt $*	(7) *Pub $*	(8) E:*Pay*	(9) Pr:*Ineff*	(10) E:*Subs*
1.1	2:1	0.12	1.4	**1.12**	**25.0%**	**1.1111**	**1.4815**	**1.1111**	**0.000**	**0.000**
1.2	2:1	0.12	1.4	1.11	26.1%	1.1113	1.4830	1.1103	0.003	0.001
1.3	2:1	0.12	1.4	1.08	29.6%	1.1226	1.5332	1.0909	0.091	0.032
1.4	2:1	0.12	1.4	1.05	33.3%	1.1430	1.5972	1.0741	0.183	0.069
1.5	2:1	0.12	1.4	1.00	40.0%	1.1929	1.7239	1.0526	0.333	0.140
2.1	2:1	0.12	1.3	***1.12***	***16.1%***	1.1111	***1.4815***	***1.1111***	***0.000***	***0.000***
2.2	2:1	0.12	1.3	1.11	17.1%	1.1110	1.4822	1.1103	0.003	0.001
2.3	2:1	0.12	1.3	**1.10**	**18.2%**	**1.1106**	1.4902	**1.1036**	**0.032**	**0.007**
2.4	2:1	0.12	1.3	1.09	19.3%	1.1111	1.4998	1.0971	0.061	0.014
2.5	2:1	0.12	1.3	1.08	20.4%	1.1127	1.5109	1.0909	0.091	0.022
2.6	2:1	0.12	1.3	1.07	21.5%	1.1153	1.5235	1.0850	0.122	0.030
2.7	2:1	0.12	1.3	1.05	23.8%	1.1233	1.5527	1.0741	0.183	0.049
2.8	2:1	0.12	1.3	1.00	30.0%	1.1578	1.6438	1.0526	0.333	0.105
3.1	2:1	0.12	1.2	1.12	7.1%	1.1111	1.4815	1.1111	0.000	0.000
3.2	2:1	0.12	1.2	***1.11***	***8.1%***	1.1106	***1.4814***	***1.1103***	***0.003***	***0.000***
3.3	2:1	0.12	1.2	1.10	9.1%	1.1071	1.4824	1.1036	0.032	0.003
3.4	2:1	0.12	1.2	1.08	11.1%	1.1028	1.4886	1.0909	0.091	0.012
3.5	2:1	0.12	1.2	**1.07**	**12.1%**	**1.1021**	1.4938	**1.0850**	**0.122**	**0.017**
3.6	2:1	0.12	1.2	1.06	13.2%	1.1024	1.5004	1.0794	0.152	0.023
3.7	2:1	0.12	1.2	1.04	15.4%	1.1057	1.5171	1.0691	0.214	0.037
3.8	2:1	0.12	1.2	1.02	17.6%	1.1127	1.5387	1.0602	0.275	0.053
3.9	2:1	0.12	1.2	1.00	20.0%	1.1228	1.5637	1.0526	0.333	0.070
4.1	2:1	0.12	1.1	1.10	0.0%	1.1036	1.4746	1.1036	0.032	0.000
4.2	2:1	0.12	1.1	1.07	2.8%	1.0890	1.4641	1.0850	0.122	0.004
4.3	2:1	0.12	1.1	***1.06***	***3.8%***	1.0860	***1.4632***	***1.0794***	***0.152***	***0.007***
4.4	2:1	0.12	1.1	1.05	4.8%	1.0839	1.4636	1.0741	0.183	0.010
4.5	2:1	0.12	1.1	1.04	5.8%	1.0828	1.4652	1.0691	0.214	0.014
4.6	2:1	0.12	1.1	**1.03**	**6.8%**	**1.0827**	1.4681	**1.0645**	**0.245**	**0.018**
4.7	2:1	0.12	1.1	1.02	7.8%	1.0835	1.4722	1.0602	0.275	0.023
4.8	2:1	0.12	1.1	1.00	10.0%	1.0877	1.4836	1.0526	0.333	0.035
5.1	2:1	0.12	1.05	1.05	0.0%	1.0741	1.4413	1.0741	0.183	0.000
5.2	2:1	0.12	1.05	1.04	1.0%	1.0714	1.4392	1.0691	0.214	0.002
5.3	2:1	0.12	1.05	***1.03***	***1.9%***	1.0697	***1.4385***	***1.0645***	***0.245***	***0.005***
5.4	2:1	0.12	1.05	**1.02**	**2.9%**	**1.0690**	1.4390	**1.0602**	**0.275**	**0.009**
5.5	2:1	0.12	1.05	1.01	4.0%	1.0692	1.4408	1.0563	0.305	0.013
5.6	2:1	0.12	1.05	1.00	5.0%	1.0701	1.4435	1.0526	0.333	0.018
6.1	2:1	0.12	1.02	1.02	0.0%	1.0602	1.4191	1.0602	0.275	0.000
6.2	2:1	0.12	1.02	**1.01**	**1.0%**	**1.0595**	**1.4188**	**1.0563**	**0.305**	**0.003**
6.3	2:1	0.12	1.02	1.00	2.0%	1.0596	1.4195	1.0526	0.333	0.007

Table 4

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
ID	Com	*u*	*Cd*	*c*	*s*	*Pvt $*	*Pub $*	E:*Pay*	Pr:*Ineff*	E:*Subs*
1.1	2:1	0.062	1.2	**1.06**	**13.2%**	**1.0526**	**1.4035**	**1.053**	**0.000**	**0.000**
1.2	2:1	0.062	1.2	1.05	14.3%	1.0535	1.4078	1.051	0.016	0.003
1.3	2:1	0.062	1.2	1.04	15.4%	1.0578	1.4263	1.045	0.080	0.013
1.4	2:1	0.062	1.2	1.02	17.6%	1.0728	1.4724	1.034	0.210	0.039
1.5	2:1	0.062	1.2	1.00	20.0%	1.0940	1.5253	1.026	0.333	0.068
2.1	2:1	0.12	1.2	1.12	7.1%	1.1111	1.4815	1.1111	0.000	0.000
2.2	2:1	0.12	1.2	***1.11***	***8.1%***	1.1106	***1.4814***	***1.1103***	***0.003***	***0.000***
2.3	2:1	0.12	1.2	1.10	9.1%	1.1071	1.4824	1.1036	0.032	0.003
2.4	2:1	0.12	1.2	1.08	11.1%	1.1028	1.4886	1.0909	0.091	0.012
2.5	2:1	0.12	1.2	**1.07**	**12.1%**	**1.1021**	1.4938	**1.0850**	**0.122**	**0.017**
2.6	2:1	0.12	1.2	1.06	13.2%	1.1024	1.5004	1.0794	0.152	0.023
2.7	2:1	0.12	1.2	1.03	16.5%	1.1089	1.5275	1.0646	0.245	0.044
2.8	2:1	0.12	1.2	1.00	20.0%	1.1228	1.5637	1.0526	0.333	0.070
3.1	2:1	0.28	1.2	1.20	0.0%	1.2432	1.6820	1.2432	0.122	0.000
3.2	2:1	0.28	1.2	1.12	7.1%	1.2171	1.6638	1.1974	0.205	0.020
3.3	2:1	0.28	1.2	***1.11***	***8.1%***	1.2154	***1.6637***	***1.1922***	***0.216***	***0.023***
3.4	2:1	0.28	1.2	1.10	9.1%	1.2139	1.6639	1.1870	0.227	0.027
3.5	2:1	0.28	1.2	1.08	11.1%	1.2123	1.6660	1.1772	0.248	0.035
3.6	2:1	0.28	1.2	**1.07**	**12.1%**	**1.2120**	1.6678	**1.1725**	**0.259**	**0.039**
3.7	2:1	0.28	1.2	1.06	13.2%	1.2121	1.6702	1.1680	0.270	0.044
3.8	2:1	0.28	1.2	1.00	20.0%	1.2191	1.6921	1.1429	0.333	0.076
4.1	2:1	0.52	1.2	1.20	0.0%	1.4339	1.96107	1.434	0.2460	0.000
4.2	2:1	0.52	1.2	1.11	8.1%	1.4213	1.95202	1.386	0.2847	0.036
4.3	2:1	0.52	1.2	***1.10***	***9.1%***	1.4206	***1.95197***	***1.381***	***0.2891***	***0.040***
4.4	2:1	0.52	1.2	1.09	10.1%	1.4201	1.95219	1.376	0.2935	0.044
4.5	2:1	0.52	1.2	1.07	12.1%	1.4196	1.95323	1.366	0.3023	0.054
4.6	2:1	0.52	1.2	**1.06**	**13.2%**	**1.4195**	1.95406	**1.361**	**0.3067**	**0.058**
4.7	2:1	0.52	1.2	1.04	15.4%	1.4199	1.95625	1.352	0.3156	0.068
4.8	2:1	0.52	1.2	1.00	20.0%	1.4222	1.96290	1.333	0.3333	0.089

Table 5

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
ID	Com	*u*	*Cd*	*c*	*s*	*Pvt $*	*Pub $*	E:*Pay*	Pr:*Ineff*	E:*Subs*
1.1	2:1	0.12	1.2	1.12	7.1%	1.1111	1.4815	1.1111	0.000	0.000
1.2	2:1	0.12	1.2	***1.11***	***8.1%***	1.1106	***1.4814***	***1.1103***	***0.003***	***0.000***
1.3	2:1	0.12	1.2	1.10	9.1%	1.1071	1.4824	1.1036	0.032	0.003
1.4	2:1	0.12	1.2	1.08	11.1%	1.1028	1.4886	1.0909	0.091	0.012
1.5	2:1	0.12	1.2	**1.07**	**12.1%**	**1.1021**	1.4938	**1.0850**	**0.122**	**0.017**
1.6	2:1	0.12	1.2	1.06	13.2%	1.1024	1.5004	1.0794	0.152	0.023
1.7	2:1	0.12	1.2	1.04	15.4%	1.1057	1.5171	1.0691	0.214	0.037
1.8	2:1	0.12	1.2	1.02	17.6%	1.1127	1.5387	1.0602	0.275	0.053
1.9	2:1	0.12	1.2	1.00	20.0%	1.1228	1.5637	1.0526	0.333	0.070
2.1	3:1	0.12	1.2	**1.06**	**13.2%**	**1.0526**	**1.4035**	**1.0526**	**0.000**	**0.000**
2.2	3:1	0.12	1.2	1.04	15.4%	1.0584	1.4244	1.0474	0.066	0.011
2.3	3:1	0.12	1.2	1.02	17.6%	1.0707	1.4602	1.0402	0.163	0.031
2.4	3:1	0.12	1.2	1.00	20.0%	1.0862	1.4983	1.0345	0.250	0.052
3.1	2:1	0.12	1.1	1.10	0.0%	1.1036	1.4746	1.1036	0.032	0.000
3.2	2:1	0.12	1.1	1.07	2.8%	1.0890	1.4641	1.0850	0.122	0.004
3.3	2:1	0.12	1.1	***1.06***	***3.8%***	1.0860	***1.4632***	***1.0794***	***0.152***	***0.007***
3.4	2:1	0.12	1.1	1.05	4.8%	1.0839	1.4636	1.0741	0.183	0.010
3.5	2:1	0.12	1.1	1.04	5.8%	1.0828	1.4652	1.0691	0.214	0.014
3.6	2:1	0.12	1.1	**1.03**	**6.8%**	**1.0827**	1.4681	**1.0645**	**0.245**	**0.018**
3.7	2:1	0.12	1.1	1.02	7.8%	1.0835	1.4722	1.0602	0.275	0.023
3.8	2:1	0.12	1.1	1.00	10.0%	1.0877	1.4836	1.0526	0.333	0.035
4.1	3:1	0.12	1.1	***1.06***	***3.8%***	1.0526	***1.4035***	***1.053***	***0.000***	***0.000***
4.2	3:1	0.12	1.1	1.05	4.8%	1.0522	1.4044	1.052	0.014	0.001
4.3	3:1	0.12	1.1	**1.04**	**5.8%**	**1.0515**	1.4086	**1.047**	**0.066**	**0.004**
4.4	3:1	0.12	1.1	1.03	6.8%	1.0520	1.4143	1.044	0.116	0.008
4.5	3:1	0.12	1.1	1.02	7.8%	1.0538	1.4213	1.04	0.163	0.014
4.6	3:1	0.12	1.1	1.00	10.0%	1.0604	1.4388	1.035	0.250	0.026
5.1	4:1	0.12	1.1	**1.04**	**5.8%**	**1.0345**	**1.3793**	**1.035**	**0.000**	**0.000**
5.2	4:1	0.12	1.1	1.03	6.8%	1.0352	1.3832	1.033	0.029	0.002
5.3	4:1	0.12	1.1	1.02	7.8%	1.0378	1.3928	1.03	0.091	0.007
5.4	4:1	0.12	1.1	1.00	10.0%	1.0461	1.4148	1.026	0.200	0.021

Table 6

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
ID	Com	*u*	*Cd*	*c*	*s*	*Pvt $*	*Pub $*	E:*Pay*	Pr:*Ineff*	E:*Subs*
1.1	1:2	0.12	1.4	1.333	5.0%	1.3800	1.9505	1.3551	0.276	0.025
1.2	1:2	0.12	1.4	1.176	19.0%	1.3038	1.8868	1.2039	0.371	0.100
1.3	1:2	0.12	1.4	***1.163***	***20.4%***	1.2995	***1.8863***	***1.1910***	***0.384***	***0.108***
1.4	1:2	0.12	1.4	1.149	21.8%	1.2960	1.8870	1.1786	0.398	0.117
1.5	1:2	0.12	1.4	1.111	26.0%	1.2901	1.8974	1.1437	0.443	0.146
1.6	1:2	0.12	1.4	**1.099**	**27.4%**	**1.2898**	1.9039	**1.1327**	**0.460**	**0.157**
1.7	1:2	0.12	1.4	1.087	28.8%	1.2904	1.9120	1.1223	0.479	0.168
1.8	1:2	0.12	1.4	1.053	33.0%	1.2982	1.9470	1.0931	0.540	0.205
1.9	1:2	0.12	1.4	1.000	40.0%	1.3333	2.0444	1.0526	0.667	0.281
2.1	1:2	0.12	1.2	1.190	0.8%	1.2213	1.7002	1.2171	0.359	0.004
2.2	1:2	0.12	1.2	1.149	4.4%	1.2023	1.6826	1.1786	0.398	0.024
2.3	1:2	0.12	1.2	1.111	8.0%	1.1887	1.6736	1.1437	0.443	0.045
2.4	1:2	0.12	1.2	***1.099***	***9.2%***	1.1855	***1.6727***	***1.1327***	***0.460***	***0.053***
2.5	1:2	0.12	1.2	1.087	10.4%	1.1830	1.6731	1.1223	0.479	0.061
2.6	1:2	0.12	1.2	1.075	11.6%	1.1812	1.6745	1.1122	0.498	0.069
2.7	1:2	0.12	1.2	**1.053**	**14.0%**	**1.1801**	1.6815	**1.0931**	**0.540**	**0.087**
2.8	1:2	0.12	1.2	1.042	15.2%	1.1808	1.6870	1.0842	0.563	0.097
2.9	1:2	0.12	1.2	1.000	20.0%	1.1930	1.7239	1.0526	0.667	0.140
3.1	1:4	0.12	1.2	1.190	0.8%	1.2071	1.6840	1.2029	0.372	0.004
3.2	1:4	0.12	1.2	1.124	6.8%	1.1774	1.6602	1.1381	0.452	0.039
3.3	1:4	0.12	1.2	***1.111***	***8.0%***	1.1733	***1.6587***	***1.1261***	***0.472***	***0.047***
3.4	1:4	0.12	1.2	1.099	9.2%	1.1702	1.6589	1.1146	0.493	0.056
3.5	1:4	0.12	1.2	1.075	11.6%	1.1662	1.6633	1.0924	0.542	0.074
3.6	1:4	0.12	1.2	**1.064**	**12.8%**	**1.1656**	1.6681	**1.0817**	**0.570**	**0.084**
3.7	1:4	0.12	1.2	1.053	14.0%	1.1662	1.6748	1.0715	0.600	0.095
3.8	1:4	0.12	1.2	1.000	20.0%	1.1897	1.7463	1.0256	0.800	0.164

Table 7

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
ID	Com	*u*	*Cd*	*c*	*s*	*Pvt $*	*Pub $*	E:*Pay*	Pr:*Ineff*	E:*Subs*
1.1	1:2	0.12	1.2	1.190	0.8%	1.2213	1.7002	1.2171	0.359	0.004
1.2	1:2	0.12	1.2	1.149	4.4%	1.2023	1.6826	1.1786	0.398	0.024
1.3	1:2	0.12	1.2	1.111	8.0%	1.1887	1.6736	1.1437	0.443	0.045
1.4	1:2	0.12	1.2	***1.099***	***9.2%***	***1.1855***	***1.6727***	***1.1327***	***0.460***	***0.053***
1.5	1:2	0.12	1.2	1.087	10.4%	1.1830	1.6731	1.1223	0.479	0.061
1.6	1:2	0.12	1.2	1.075	11.6%	1.1812	1.6745	1.1122	0.498	0.069
1.7	1:2	0.12	1.2	**1.053**	**14.0%**	**1.1801**	**1.6815**	**1.0931**	**0.540**	**0.087**
1.8	1:2	0.12	1.2	1.042	15.2%	1.1808	1.6870	1.0842	0.563	0.097
1.9	1:2	0.12	1.2	1.000	20.0%	1.1930	1.7239	1.0526	0.667	0.140
2.1	1:2	0.28	1.2	1.1111	8.0%	1.2961	1.84193	1.2337	0.5689	0.062
2.2	1:2	0.28	1.2	***1.0989***	***9.2%***	***1.2947***	***1.84189***	***1.2232***	***0.5781***	***0.071***
2.3	1:2	0.28	1.2	1.0870	10.4%	1.2936	1.84227	1.2131	0.5874	0.081
2.4	1:2	0.28	1.2	1.0638	12.8%	1.2923	1.84430	1.1937	0.6064	0.099
2.5	1:2	0.28	1.2	**1.0526**	**14.0%**	**1.2920**	**1.84593**	**1.1845**	**0.6162**	**0.108**
2.6	1:2	0.28	1.2	1.0417	15.2%	1.2921	1.84800	1.1756	0.6260	0.117
2.7	1:2	0.28	1.2	1.0000	20.0%	1.2952	1.86032	1.1429	0.6667	0.152
3.1	1:2	0.36	1.2	1.1111	8.0%	1.3621	1.93509	1.2937	0.5949	0.068
3.2	1:2	0.36	1.2	***1.0989***	***9.2%***	***1.3610***	***1.93506***	***1.2830***	***0.6018***	***0.078***
3.3	1:2	0.36	1.2	1.0870	10.4%	1.3602	1.93531	1.2726	0.6088	0.088
3.4	1:2	0.36	1.2	1.0526	14.0%	1.3587	1.93764	1.2433	0.6301	0.115
3.5	1:2	0.36	1.2	**1.0417**	**15.2%**	**1.3586**	**1.93893**	**1.2341**	**0.6373**	**0.125**
3.6	1:2	0.36	1.2	1.0309	16.4%	1.3587	1.94049	1.2252	0.6446	0.134
3.7	1:2	0.36	1.2	1.0000	20.0%	1.3600	1.94667	1.2000	0.6667	0.160
4.1	1:2	0.52	1.2	1.1111	8.0%	1.51479	2.14421	1.4354	0.6224	0.079
4.2	1:2	0.52	1.2	***1.0989***	***9.2%***	***1.51410***	***2.14416***	***1.4239***	***0.6268***	***0.090***
4.3	1:2	0.52	1.2	1.0870	10.4%	1.51349	2.14422	1.4127	0.6312	0.101
4.4	1:2	0.52	1.2	1.0526	14.0%	1.51204	2.14495	1.3809	0.6445	0.131
4.5	1:2	0.52	1.2	1.0101	18.8%	1.51115	2.14731	1.3423	0.6622	0.169
4.6	1:2	0.52	1.2	**1.0000**	**20.0%**	**1.51111**	**2.14814**	**1.3333**	**0.6667**	**0.178**
4.7	1:2	0.52	1.2	0.9901	21.2%	1.51115	2.14909	1.3246	0.6711	0.187

Table 8

Private	Bidtaker's		Expected Cost		(*Pvt $*)
	Cd				
Subsidy:	1.20	1.18	1.15	1.12	1.10
0%	1.2609	1.2435	1.2180	1.1936	1.1780
15%	1.2181	1.2082	1.1946	1.1826	1.1755
Full Info:	1.2181	1.2078	1.1921	1.1763	1.1656
Benefit	100.0%	98.9%	90.3%	63.6%	20.2%
Public	Bidtaker's		Expected Cost		(*Pub $*)
	Cd				
Subsidy:	1.20	1.18	1.15	1.12	1.10
0%	1.746	1.7189	1.6778	1.6372	1.6105
11%	1.711	1.6902	1.6592	1.6289	1.6091
Full Info:	1.711	1.6899	1.6571	1.6235	1.6008
Benefit	100.0%	99.0%	89.9%	60.6%	14.4%

Table 9

Private	Bidtaker's		Expected Cost		(*Pvt $*)
	u				
Subsidy:	0.28	0.19	0.062	0.013	0.0013
0%	1.4843	1.3455	1.2171	1.2007	1.2000
17.2%	1.4663	1.3186	1.1501	1.0767	1.0313
Full Info:	1.4663	1.3184	1.1481	1.0747	1.0267
Benefit	100.0%	99.3%	97.0%	98.4%	97.4%

Public	Bidtaker's		Expected Cost		(*Pub $*)
	u				
Subsidy:	0.28	0.19	0.062	0.013	0.0013
0%	2.0659	1.8728	1.6665	1.6122	1.6012
12.8%	2.0518	1.8513	1.6059	1.4759	1.4252
Full Info:	2.0518	1.8512	1.6053	1.4717	1.3813
Benefit	100.0%	99.4%	99.0%	97.0%	80.0%